ABSTRACT: Auditors may encounter misstatements during the course of an audit, each of which requires a binary materiality assessment. We propose a fuzzy expert system approach that assesses materiality as a continuous characteristic by allowing a misstatement to possess a degree of materiality between 0 and 1. This potentially allows the auditor more flexibility and precision in materiality assessment, and greater insight regarding subsequent testing and investigation. We demonstrate that a fuzzy expert system can help the auditor incorporate qualitative factors into the materiality assessment of each misstatement and identify which misstatements are most worthy of further investigation. The auditor may compare the materiality assessments of all misstatements to plan an audit strategy. By providing a formal model structure, the fuzzy expert system formalizes and documents the materiality assessment process. This may facilitate better communication within the audit team and with the client, and enhances process consistency across auditors, engagements, and years.

Keywords: fuzzy logic; materiality; auditing; expert systems.

INTRODUCTION

The Financial Accounting Standards Board (FASB 1980, 132) states:

The omission or misstatement of an item in a financial report is material if, in light of surrounding circumstance, the magnitude of the item is such that it is probable that the judgment of a reasonable person relying upon the report would have been changed or influenced by the inclusion or correction of the item.

In practice, the auditor must make a binary decision for each omission and misstatement: it is either material, or it is not. This stark contrast divides the set of all omissions and misstatements into two subsets: those that are material, and those that are not. We believe that this is an oversimplification. We propose an alternative model that uses fuzzy logic, in which omissions and misstatements possess a degree of membership in the set of all material omissions and misstatements. Put another way, each omission or misstatement is material to a greater or lesser degree, measured on a scale from 0 to 1.
In a simple example, consider the set $T = \{x \mid x \text{ is a tall man}\}$. Classical set theory imposes a binary structure in which we must establish a threshold, say six feet, and declare that a man is a member of $T$ if and only if he is at least six feet tall. We might well be concerned about declaring a man to be tall if he is six feet tall but not if he is five feet and 15/16th inches tall. Do we really mean to be so arbitrary? Using the fuzzy logic approach, we might assign membership equal to 0 to a man who is five feet tall, membership equal to 1 to a man who is seven feet tall, and membership equal to 0.5 to a man who is six feet tall.

The purpose of this paper is to demonstrate the application of fuzzy logic to assessing materiality. Our idea is to build an expert system based on fuzzy logic that assesses materiality. This approach has several advantages.

First, treating materiality as a fuzzy concept provides the auditor with greater discriminatory power relative to that available using the classical binary notion. By assigning a degree of membership in the set of material misstatements and omissions, the fuzzy expert system advises the auditor regarding the significance of a misstatement or omission and creates a materiality rating for each. This allows the auditor to focus attention on the items of greatest significance, thereby enhancing the overall efficiency of the audit process.

Second, the fuzzy expert system allows for explicit consideration of important qualitative factors relevant to materiality. This contrasts with the commonly used quantitative rules of thumb, such as 5 percent of net income, for determining materiality, which ignore qualitative factors.

Third, the fuzzy expert system provides a model structure that requires the auditor to make explicit judgments, thereby formalizing and documenting the materiality assessment process. This may allow for better communication within the audit team and with the client, and enhance process consistency across auditors, engagements, and years.

Fourth, the fuzzy expert system establishes a framework for organizational learning. When incorporated with a formal feedback system, such as those found in neural networks, fuzzy expert systems can learn from their mistakes and improve future performance by adjusting its parameters appropriately. Thus, if materiality assessments are found later to be inaccurate, the fuzzy expert system can modify itself to improve future assessments of similar situations.

Our fuzzy logic approach contrasts with both the Bayesian (Box and Tiao 1973) and the Dempster-Shafer (Dempster 1968; Shafer 1976) approaches to evaluating the strength of evidence. The Bayesian approach establishes a prior probability distribution for a relevant random variable and updates the distribution based on observed data. In assessing materiality, the auditor would establish a prior probability that a misstatement is material using all available information about the firm, the industry in which it operates, and other relevant facts other than the misstatement itself. The auditor then updates that probability based on the specific quantitative and qualitative characteristics of the misstatement.

The Dempster-Shafer approach assigns belief probabilities to propositions and uses a formal mechanism for combining evidence concerning these propositions from two or more sources. This allows an auditor to combine multiple disparate observations concerning a misstatement into an overall belief that the misstatement is material. The approach also allows for the calculations of the plausibility that the misstatement is not material, which may serve as a useful indicator of the adequacy of the total evidence collected.

Both the Bayesian and the Dempster-Shafer approaches are founded in probability theory while the fuzzy logic approach uses membership functions that resemble probabilities but do have the same mathematical properties. Membership functions serve as a mechanism for expressing a degree of membership of an element in a set rather than the probability that the element is in the set. Thus, as

---

1 In some cases, a simple arbitrary cutoff may well serve the purpose, as in deciding whether a piece of luggage qualifies as carry-on luggage. If the piece exceeds a certain dimension, it simply will not fit into the overhead compartment.
A Fuzzy Logic Approach to Assessing Materiality

The next section provides background on materiality assessment and discusses some existing applications of fuzzy logic in auditing. The subsequent sections briefly cover the fundamentals of fuzzy sets, fuzzy logic, and the fuzzy inference process, and describe how these concepts apply to materiality assessment. The final sections present a prototype of a fuzzy expert system for assessing materiality, and conclude with a brief discussion and directions for future research.

BACKGROUND

Assessing Materiality

Auditors tend to view materiality as a quantitative concept—the larger the fluctuation, the more likely the auditor is to consider it material. This is natural since size is easier to measure and analyze than nonquantitative factors. However, both the SEC and the FASB recognize that such thresholds and rules of thumb can be useful starting points, but that “exclusive reliance on (numerical thresholds) has no basis in accounting literature or law.” Indeed, materiality assessment requires the consideration of many qualitative factors beyond the size of the misstatement or omission.2 The SEC (1999) Staff Accounting Bulletin No. 99 lists several qualitative factors that “render material a quantitatively small misstatement of a financial statement item.” These include:

- whether the misstatement arises from an item capable of precise measurement or whether it arises from an estimate and, if so, the degree of imprecision inherent in the estimate
- whether the misstatement masks a change in earnings or other trends
- whether the misstatement hides a failure to meet analysts’ consensus expectations for the enterprise
- whether the misstatement changes a loss into income or vice versa
- whether the misstatement concerns a segment or other portion of the registrant’s business that has been identified as playing a significant role in the registrant’s operations or profitability
- whether the misstatement affects the registrant’s compliance with regulatory requirements
- whether the misstatement affects the registrant’s compliance with loan covenants or other contractual requirements
- whether the misstatement has the effect of increasing management’s compensation—for example, by satisfying requirements for the award of bonuses or other forms of incentive compensation
- whether the misstatement involves concealment of an unlawful transaction.

The FASB has also emphasized that materiality is not strictly a quantitative concept (FASB 1980). In fact, they rejected a formulaic approach for determining materiality in favor of one that incorporates all relevant circumstances.

Typically, qualitative factors are more difficult to assess than the size of the misstatement or omission. Qualitative factors often require subjective judgment and evaluation in light of other information that may not be readily available to the auditor during the audit, such as whether the misstatement or omission hides the failure to meet analysts’ consensus expectations for the enterprise. Qualitative factors may also require considerable effort to evaluate properly.

Consequently, auditors tend to rely on quantitative evaluations and do so using simplistic numerical thresholds and rules of thumb. We reviewed the materiality worksheets of three national public accounting firms and found no specific guidance regarding the evaluation of qualitative factors in the materiality assessment process. One worksheet made no mention at all of qualitative factors. A second worksheet reminded the auditor to consider such factors but only provided a single example, that of an illegal payment. The third worksheet listed the qualitative factors from

---

2 We consider misstatements, in which a numerical or textual item is reported incorrectly, and omissions, in which a required item is absent from the financial statement. Both of these are errors, which may or may not be material.
SEC (1999) Staff Accounting Bulletin No. 99, but provided no methodology for incorporating such factors into the overall materiality assessment. Thus, we believe that qualitative factors, while recognized as important, are likely to be overlooked.

Existing Applications of Fuzzy Logic to Accounting and Business


Friedlob and Schleifer (1999) describe how to apply fuzzy logic in several audit situations involving risk and uncertainty. Their paper serves as an introduction to the basic concepts of fuzzy logic and provides examples of how fuzzy logic applies to the audit risk model. They present fuzzy logic as an alternative paradigm to probability in this context. Lenard et al. (2000) uses fuzzy logic to model the auditor’s going concern decision. They construct two models, one based entirely on a fuzzy clustering algorithm and one that is a hybrid system based on the M-estimator. While their fuzzy clustering model produces results that are slightly less accurate than those produced by the hybrid system, their fuzzy approach identifies additional clusters. This additional information about the structure of the data may assist auditors in recognizing that there may be a going-concern problem even if one or more of the financial variables indicates otherwise.

Applying Fuzzy Logic to Materiality Assessment

We present below a prototype of a fuzzy expert system that captures many of the considerations required in judging materiality. We stress that the following is strictly a prototype and we make no claim that this system is either complete or necessarily optimal. See the Appendix for a discussion of the fundamentals of fuzzy logic.

Defining the Fuzzy Sets and Determining Memberships

Consider an auditor who is evaluating the materiality of a misstatement in the client’s estimate of uncollectible accounts. The auditor might first examine the quantitative nature of the misstatement, namely the size of the misstatement, using whatever measure (total revenue, total assets, etc.) the auditor deems appropriate, and the precision with which it has been measured.

Suppose the auditor identifies five fuzzy sets for the size of the misstatement, and calls the fuzzy sets Very Small, Small, Medium, Large, and Very Large. The auditor then assigns membership values for the misstatement in each of these five fuzzy sets. These membership values must be nonnegative and are often chosen to sum to 1, although this is not necessary. The auditor might also introduce five fuzzy sets for the precision with which the misstatement is measured, and call them Very Low, Low, Moderate, High, and Very High. Similarly, the auditor then assigns membership values for the precision in each of these five fuzzy sets. Again, these membership values must be nonnegative. Suppose the selected memberships assignments for size and precision are those shown in Table 1.
TABLE 1

Membership Assignments for the Size of the Misstatement and the Precision with Which It Is Measured in the Example

<table>
<thead>
<tr>
<th>Quantitative Factor</th>
<th>Size of the misstatement</th>
<th>Precision with which the misstatement is measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Small</td>
<td>Small 0.3</td>
<td>Medium 0.7</td>
</tr>
<tr>
<td>Small</td>
<td>Very Low</td>
<td>Low 0.6</td>
</tr>
<tr>
<td>Medium</td>
<td>Moderate 0.4</td>
<td>High</td>
</tr>
<tr>
<td>Large</td>
<td>Very High</td>
<td></td>
</tr>
<tr>
<td>Very Large</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observe that, in evaluating the size of the misstatement concerning uncollectible accounts, the auditor assigns memberships of 0.3 for Small and 0.7 for Medium. This captures the auditor’s opinion that the size of the misstatement is somewhere between small and medium, but somewhat closer to medium. Similarly, observe that, in evaluating the precision with which the misstatement is measured, the auditor assigns memberships of 0.6 for Low and 0.4 for Moderate. This captures the auditor’s opinion that the precision is somewhere between low and moderate but somewhat closer to low.

We acknowledge that replacing a continuous variable with a categorical variable results in loss of information. In this regard, we are tempted to use the size of the misstatement and the precision to which it is measured as continuous variables. However, we believe that the auditor considers other factors when evaluating the size and precision of the misstatement. For example, a $50,000 misstatement in a small firm will be considered more material than would the same misstatement in a large firm. In addition, within a firm, a $50,000 misstatement in an inventory account may be considered more or less material than would the same misstatement in a payroll account.

The first issue might be resolved by normalizing the size of the misstatement using some appropriate measure of firm size, such as total assets or total revenues. However, even if there were agreement on the choice of normalizing measure, we would still face the second issue. In short, the auditor employs judgment involving the context of the misstatement even when evaluating the materiality of a continuously measured variable. The important information, therefore, is a combination of the continuous variable and the contextual judgment of the auditor, which we choose to express on a categorical scale.

Next, suppose the auditor defines five fuzzy sets for each of the qualitative factors listed earlier, and calls them Not at All, Slightly, Moderately, Considerably, and Completely. The auditor then assigns membership values for each in these five fuzzy sets. Again, these membership values must be nonnegative. Suppose the selected membership assignments for the qualitative factors are those shown in Table 2. Observe that the auditor assigns memberships of 0.5, 0.3, and 0.2 to the fuzzy sets Not at All, Slightly, and Moderately, respectively, for the extent to which the misstatement in the client’s estimate of uncollectible accounts masks changes in earnings. This captures the auditor’s opinion that the misstatement does little to mask changes in earnings. Similar interpretations follow for the remaining qualitative factors.

We make no claim that different auditors will provide the same membership evaluations when examining the same misstatement. One advantage of the fuzzy expert system is that it clarifies specifically where disagreements take place, thereby allowing the auditors to focus on the reasons for their specific variations. Thus, the overall effect of the system is to reduce the extent to which different auditors produce different materiality assessments.

The Fuzzy Rules

Assume that the auditor establishes fuzzy rules of the following types:

A. IF (Size is X AND Precision is Y) THEN (Misstatement is Material), and
B. IF (Qualitative Factor is Z) THEN (Misstatement is Material)

where X, Y, and Z represent the names of the fuzzy sets associated with the corresponding factors.
TABLE 2
Membership Assignments for the Qualitative Factors in the Example

<table>
<thead>
<tr>
<th>Qualitative Factor</th>
<th>Not at All</th>
<th>Slightly</th>
<th>Moderately</th>
<th>Considerably</th>
<th>Completely</th>
</tr>
</thead>
<tbody>
<tr>
<td>masks a change in earnings or other trends</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hides a failure to meet analysts’ consensus expectations for the enterprise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>changes a loss into income or vice versa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concerns a segment or other portion of the registrant’s business that has been identified as playing a significant role in the registrant’s operations or profitability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>affects the registrant’s compliance with regulatory requirements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>affects the registrant’s compliance with loan covenants or other contractual requirements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>has the effect of increasing management’s compensation</td>
<td>0.95</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>involves concealment of an unlawful transaction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There are 25 rules of Type A, one for each combination of the five sizes and the five levels of precision. There are 40 rules of Type B, one for each of the five fuzzy sets associated with each of the eight qualitative factors. Each misstatement belongs to the fuzzy set Material to some degree between 0 and 1.

We point out that, even though this expert system contains 65 rules, the auditor needs to make only ten judgments, one each for size and precision, and one for each of the qualitative factors. In general, the expert system may contain hundreds of rules while requiring only a small number of auditor assessments.

Assessing Validities

Next, the auditor establishes validities for the 25 rules concerning size and precision and for the 40 qualitative rules. See Tables 3 and 4. (The validities presented here are for illustration only and may not reflect appropriate values in all situations.) For each of the 65 rules, the membership of the misstatement in the fuzzy set Material equals the product of the validity of the rule times the truth-value of the antecedent of the rule. The truth-value of a statement represents the degree to which the builder of the expert system considers the statement true. For example, the rule that states:

IF (Size is Large AND Precision is Very Low) THEN (Misstatement is Material)
has 0.35 membership in the set V of valid rules.

By contrast, the rule that states:

IF (Size is Medium AND Precision is High) THEN (Misstatement is Material)
has 0.90 membership in the set V of valid rules.
Thus, the auditor is expressing the opinion that a medium-sized misstatement measured with high precision provides greater evidence of materiality than does a large misstatement measured with very low precision.3

<table>
<thead>
<tr>
<th>Qualitative Factor</th>
<th>Not at All</th>
<th>Slightly</th>
<th>Moderately</th>
<th>Considerably</th>
<th>Completely</th>
</tr>
</thead>
<tbody>
<tr>
<td>masks earnings changes</td>
<td>0</td>
<td>0.85</td>
<td>0.9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>hides failure to meet analysts’ expectations</td>
<td>0</td>
<td>0.8</td>
<td>0.9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>changes a loss into income or vice versa</td>
<td>0</td>
<td>0.85</td>
<td>0.9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>concerns a significant segment of the firm</td>
<td>0</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>affects regulatory compliance</td>
<td>0</td>
<td>0.95</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>affects loan or contract compliance</td>
<td>0</td>
<td>0.9</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>increases management compensation</td>
<td>0</td>
<td>0.7</td>
<td>0.9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>conceals an unlawful transaction</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

For example, the rule that states, “a misstatement that slightly increases management compensation is material” has 70 percent membership in the set $V$ of valid rules.

3 We selected the validities in Table 3 to approximate the one-tailed p-values from the sampling distribution of the sample mean. We chose values for size that were representative of the fuzzy set descriptors (Very Small, Small, etc.) and we selected sample sizes that were representative of the fuzzy set descriptors for precision (Very Low, Low, etc.). Thus, each validity is roughly the probability of obtaining a misstatement of the stated size, or larger, in a sample with the indicated precision. However, we stress that the auditor should assign values that represent his or her assessments of the validities of the rules, and that we used this procedure simply as a convenient device for illustration purposes only.
Similar interpretations apply to the validities of the qualitative factor rules. For example, the rule that states:

IF (The misstatement moderately masks earnings changes) THEN (The misstatement is material) has 0.90 membership in the set V of valid rules, indicating equal evaluation of this situation with that of a medium-sized misstatement measured with high precision.

By contrast, the rule that states:

IF (The misstatement slightly increases management compensation) THEN (The misstatement is material) has 0.70 membership in the set V of valid rules, indicating that it provides less evidence of materiality.4

The Truth-Value for a Rule

The truth-value for a rule involving a specific fuzzy set for size and a specific fuzzy set for precision equals the product of the corresponding memberships of size and precision shown in Table 1. For example, consider the rule that states:

IF (Size is Medium AND Precision is Low) THEN (Misstatement is Material) has truth value equal to (0.7)(0.6) = 0.42 because, in our example, size has 0.7 membership in Medium and precision has 0.6 membership in Low.

Table 5 shows the truth-values for all 25 size and precision rules in our example.

The truth-value for a qualitative rule is equal to the membership associated with the antecedent because these rules do not involve any form of logical connectors such as AND or OR. For example, the rule that states:

IF (The misstatement moderately masks earnings changes) THEN (The misstatement is material) has truth-value equal to 0.2 (refer to Table 2).

Materiality Memberships and Defuzzification

The auditor then computes the membership of the misstatement in the fuzzy set Material implied by each fuzzy rule by multiplying the truth-value of the rule by its validity. Table 6 presents the results for the size and precision rules, and Table 7 presents the results for the qualitative rules.

<table>
<thead>
<tr>
<th>Size</th>
<th>Precision</th>
<th>Very Low</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Very High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Small</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Small</td>
<td></td>
<td>0</td>
<td>0.18</td>
<td>0.12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>0</td>
<td>0.42</td>
<td>0.28</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Large</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Very Large</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Each truth-value is the product of the corresponding memberships in the size and precision fuzzy sets indicated. Refer to Table 1. For example, the rule that states, IF (Size is Medium AND Precision is Low) THEN (Misstatement is Material) has truth value equal to (0.7)(0.6) = 0.42 because, in our example, size has 0.7 membership in Medium and precision has 0.6 membership in Low.

4 We selected the values in Table 4 using our judgment for illustration purposes only.
A Fuzzy Logic Approach to Assessing Materiality

For example, the rule that states, IF (Size is Medium AND Precision is Low) THEN (Misstatement is Material) implies 0.063 membership of the misstatement in the fuzzy set Material.

Finally, the auditor computes the defuzzified materiality value of the misstatement by computing the maximum value of the 65 materiality memberships. In our example, the defuzzified materiality value is 0.64. This value arises from the considerable extent to which the misstatement concerns a significant segment of the firm. The auditor may find it informative to compute two maximum values, one for the quantitative rules and one for the qualitative rules. In our example, these values are 0.154 and 0.64, respectively, indicating that the materiality of this misstatement arises primarily from its qualitative nature rather than its size and precision.
There are alternative methods for performing such defuzzification, but the choice of the maximum is very common in applications like this one. The rationale for using the maximum is as follows. Each if-then statement “probes” the misstatement to evaluate its materiality from one specific viewpoint. A misstatement need not be material for more than one reason. For example, if 64 of the 65 “probes” found no evidence of materiality but one “probe” found a very high degree of materiality, then we would conclude that the misstatement is material. Therefore, we evaluate its materiality based on the “probe” that discovered the maximum degree of materiality.

**DISCUSSION AND CONCLUSIONS**

Auditors routinely encounter several misstatements during the course of an audit, each of which requires a binary materiality assessment, that is, the auditor classifies each misstatement as either material or not material. We propose that materiality be assessed as a continuous characteristic by allowing a misstatement to possess a degree of materiality between 0 and 1. This allows the auditor more flexibility and precision in materiality assessment, and greater insight regarding subsequent testing and investigation.

Traditionally, auditors rely on quantitative rules of thumb and pay less attention to qualitative factors. We have demonstrated that a fuzzy expert system can help the auditor incorporate qualitative factors into the materiality assessment of each misstatement and identify which misstatements are most worthy of further investigation. Indeed, our example shows that qualitative factors may be more important than quantitative factors in determining materiality. The largest membership in the fuzzy set of material misstatements is 0.64, which is associated with a qualitative factor (concerns a significant segment of the firm). The largest membership associated with quantitative factors is 0.154, arising because the misstatement is of “medium” size and is measured with “moderate” precision.

In the example presented above, the auditor’s materiality assessment of 0.64 provides the auditor with a degree of belief that the misstatement in question deserves further investigation. The auditor may compare the materiality assessments of all misstatements to plan an audit strategy. As presented here, the fuzzy expert system is designed to assess the materiality of an individual misstatement and does not attempt to provide judgment concerning the aggregation of such assessments within and across accounts.

By providing a formal model structure, the fuzzy expert system formalizes and documents the materiality assessment process. It requires the auditor to evaluate each qualitative and quantitative factor explicitly, state each rule specifically, and assign validities to each rule unambiguously. This facilitates better communication within the audit team and with the client, and enhances process consistency across auditors, engagements, and years.

To build a valid and reliable fuzzy expert system to assess materiality, auditing experts need to carefully extract the quantitative and qualitative factors that auditors should apply, the fuzzy rules they should use, and the validities of these fuzzy rules. Once the initial design is complete, we may incorporate a formal feedback system, such as those found in neural networks, to enable the fuzzy expert system to learn from its mistakes and improve its future performance. Thus, if materiality assessments later prove to be inaccurate, the fuzzy expert system can modify itself to improve future assessments of similar situations. For example, suppose the expert system places a low materiality value on a particular misstatement and later developments reveal that the misstatement should have received a higher value because it represents a failure to comply with a loan covenant. The feedback loop would increase the membership in the set of valid rules of the rule that addresses failure to comply with loan covenants.

We point out that our system differs from most expert systems in a very important way. Most expert systems are designed to *emulate* the performance of an expert. Our system is designed to
improve the performance of the expert. Therefore, validation would rely on the existence of a clear and unambiguous outcome measure for materiality assessments, which in some instances does not exist and where the best we can hope for is some degree of consensus among experts. However, our view is that the consensus may be systematically biased due to the undervaluing of qualitative factors. Therefore, if our expert system replicated the performance of experts, we would have failed to achieve our goal, which is to improve their performance.

We consider the expert system illustrated in the paper to be an elementary prototype system. It represents a system that produces materiality assessments and is suitable for testing and evaluation. We certainly do not claim that it is in any way optimal or even that it necessarily is better than current practice without further refinement, as we have no evidence to support such a claim.

**FUTURE RESEARCH**

We see future research proceeding in two stages. In the first stage, we will further refine the fuzzy expert system using a panel of experienced auditors. In the second stage, we will field test the improved system.

In the first stage, we will convene a set of experienced expert auditors and to have them review and comment on the fuzzy expert system with the goal of refining the structure and the details of the rule base to accurately reflect the decision-making processes of the auditors. Then, we will ask each auditor to make materiality assessments of a number of hypothetical misstatements and to compare their assessments with those made by the refined fuzzy expert system. We will then review the discrepancies between the auditors’ judgments and the output of the fuzzy expert system, attempting to identify the nature and sources of the differences. This will permit us to refine the system further and improve its performance. We will iterate this process until the fuzzy expert system consistently replicates the assessments of the expert auditors.

In the second stage, we will provide auditors with software that allows them to apply the fuzzy expert system during actual audit engagements. The objectives will be to evaluate the impact of the fuzzy expert system on the auditors’ materiality assessment process. We will examine the extent to which the auditors rely on the system and their perceptions of the usefulness and user-friendliness of the system. This step will allow us to detect and correct software errors, improve the user interface, and further modify the rule base to reflect insights gained in practical applications.
APPENDIX
Fundamentals of Fuzzy Logic

Zadeh (1965) was the first to introduce the concepts of fuzzy sets and fuzzy logic. He and his followers developed an axiomatic paradigm that generalizes the classical concept of a set by allowing an item to have partial membership in a set, as opposed to being either entirely in the set or entirely out of the set.

In the classical definition of a set $S$, an item, $x$, either is an element of $S$, denoted $x \in S$, or is not an element of $S$, denoted $x \notin S$. Such definitions are “crisp” in the sense that the boundaries of $S$ are sharp and membership in $S$ is unambiguous. We define a fuzzy set $S$ by its membership function $\mu(x)$, a real-valued function that specifies the degree of membership of $x$ in $S$. We restrict $\mu(x)$ to the range $0 \leq \mu(x) \leq 1$, with $\mu(x) = 1$ indicating full membership of $x$ in $S$, and $\mu(x) = 0$ indicating that $x$ has no membership in $S$.

As illustrated above by the fuzzy set $T = \{x \mid x$ is a tall man$\}$, fuzzy sets allow us to define a set in the presence of linguistic ambiguity without the need to specify an often arbitrary and distorting crisp set definition. In this example, we could define a membership function for $T$ by defining the variable $h_x$ to be the height of man $x$, in feet, and setting:

$$\mu_T(x) = \begin{cases} 
0, & \text{if } h_x < 5 \\
\frac{1}{2}(h_x - 5), & \text{if } 5 \leq h_x \leq 7 \\
1, & \text{if } h_x > 7 
\end{cases}$$

Figure 1A shows this function.

Note that we could define $T$ as a classical set by specifying $T = \{x \mid h_x \geq 6\}$. In fuzzy set notation, this is equivalent to specifying the membership function:

$$\mu(x) = \begin{cases} 
1, & \text{if } x \text{ is a man who is at least six feet tall} \\
0, & \text{otherwise} 
\end{cases}$$

In this sense, classical sets are special cases of fuzzy sets.

---

**FIGURE 1A**
Membership Function for the Set, $T$, of Tall Men

---

Journal of Emerging Technologies in Accounting, 2005
Fuzzy logic operates on fuzzy sets by defining membership functions for the complement of a fuzzy set and for the union and intersection of two fuzzy sets. If \( S \) and \( T \) are fuzzy sets with membership functions \( \mu_s(x) \) and \( \mu_t(x) \), then the membership function of \( S' \) the complement of \( S \), is:
\[
\mu_{S'}(x) = 1 - \mu_s(x)
\]
the membership function of the union \( S \cup T \) is:
\[
\mu_{S \cup T}(x) = \max\{\mu_s(x), \mu_t(x)\}
\]
and the membership function of the intersection \( S \cap T \) is:
\[
\mu_{S \cap T}(x) = \min\{\mu_s(x), \mu_t(x)\}
\]
These definitions allow us to perform the fuzzy logic versions of the classical logical functions NOT, OR, and AND, respectively. We define the truth-value of the statement “\( x \) is in \( S \)” as \( \mu_s(x) \), which leads to \( 1 - \mu_s(x) \) as the truth-value for “\( x \) is not in \( S \).” The truth-value of “\( (x \) is in \( S \)) OR (\( y \) is in \( T \))” is \( \max\{\mu_s(x), \mu_t(y)\} \), and the truth-value of “\( (x \) is in \( S \)) AND (\( y \) is in \( T \))” is \( \min\{\mu_s(x), \mu_t(y)\} \). As in classical logic, we may build fuzzy logic expressions of arbitrary complexity.

The classical implication statement \( S \rightarrow T \) is equivalent to \( S' \cup T \). If \( S \) and \( T \) are fuzzy sets, then the membership function of \( S \rightarrow T \) is, from above, \( \mu_{S \rightarrow T}(x, y) = \max\{1 - \mu_s(x), \mu_t(y)\} \).

**Fuzzy Rules**

A fuzzy rule, \( r \), is a logical implication statement of the form:
\[
r: \text{IF A, THEN } y \text{ is in } T
\]
where \( A \) is a logical expression involving fuzzy sets, \( y \) is an item of unknown value about which the fuzzy rule makes inference, and \( T \) is a fuzzy set. For example, a company executive interested in forecasting the firm’s sales for the next quarter might believe.
\[
r_f: \text{IF } \{i \text{ is low AND } c \text{ is high}\} \text{ THEN } s \text{ is high}
\]
where \( i \) is a relevant interest rate, \( c \) is a measure of consumer confidence, and \( s \) is sales for the next quarter. We must define three fuzzy sets: \( I_{low} \) for low interest rate, \( C_{high} \) for high consumer confidence, and \( S_{high} \) for high sales for next quarter.

We use \( r \) to make an inference about \( y \) with respect to \( T \). To do so, let \( \alpha_r \) be the truth-value of the antecedent statement \( A \). Then the fuzzy rule infers a degree of membership of \( y \) in \( T \) to be:
\[
\mu_t(y) = \alpha_r
\]
In other words, \( y \) is a member of \( T \) to the extent that \( A \) is true. In the sales forecasting example, suppose that \( I = 7 \) percent and \( c = 53 \) percent (the percentage of consumers who believe that the economy is strong). Suppose further that these values have memberships equal to \( \mu_{I_{low}}(7\%) = 0.4 \) and \( \mu_{C_{high}}(53\%) = 0.7 \) in the fuzzy sets \( I_{low} \) and \( C_{high} \), respectively. Then:
\[
\alpha_r = \min\{0.4, 0.7\} = 0.4
\]
is the truth-value of the antecedent to rule \( r_f \), leading to the inference that:
\[
\mu_t(y) = \alpha_r \mu_t(y) = 0.4.
\]

We also consider rules themselves to be (partial) members of the fuzzy set \( V \) of all valid rules. Thus, a fuzzy rule \( r \) has membership \( \mu_v(r) \) in \( V \). In this case, the fuzzy rule infers a degree of membership of \( y \) in \( T \) to be:
\[
\mu_t(y) = \alpha_r \mu_v(r)
\]
In this case, \( y \) is a member of \( T \) to the extent that \( A \) is true and \( r \) is a member of \( V \). In the sales forecasting example, suppose that the executive assigned fuzzy rule \( r_j \), the membership \( \mu_r (r_j) = 0.8 \) in the set \( V \) of valid rules. The rule \( r_j \) would lead to the inference \( \mu_{High}^S (i, c) = \mu_{High}^S (7\%, 53\%) = (0.4)(0.8) = 0.32 \).

**Rule-Based Fuzzy Expert Systems**

A *rule-based fuzzy expert system* consists of a collection, \( R \), of fuzzy rules together with a fuzzy set, \( V \), of valid rules and a membership function \( \mu_r (r) \) defined for all \( r \in R \). In a given situation, the fuzzy expert system makes inferences about one or more items of interest by evaluating the antecedent expressions of each rule and assigning fuzzy set memberships to the items. Because different rules may produce inferences for the same item, the fuzzy expert system may infer several different degrees of membership of an item in a given fuzzy set and with degrees of membership in several different fuzzy sets. In the sales forecasting example, the fuzzy expert system might consist of four fuzzy rules and three fuzzy sets for sales, \( S_{Low} \), \( S_{Med} \) and \( S_{High} \) for low, moderate, and high quarterly sales. The four rules might produce

\[
  r_1 : \mu_{High}^S (\cdot) = 0.32 \\
  r_2 : \mu_{Med}^S (\cdot) = 0.65 \\
  r_3 : \mu_{Low}^S (\cdot) = 0.12 \\
  r_4 : \mu_{Low}^S (\cdot) = 0.20
\]

Note that the four fuzzy rules provide degrees of membership of \( S \) in all three fuzzy sets, and that rules 3 and 4 provide two different degrees of membership in \( S_{Low} \).

In some applications, we may not need to proceed further. The executive in our example might be satisfied to interpret these degrees of membership to predict that sales in the next quarter will be moderate to high. However, in other applications, we must convert the fuzzy inference results to crisp outcomes. We refer to this last step as *defuzzification*.

To illustrate the defuzzification process in the sales forecasting example, suppose that the executive has specified the membership functions for \( S_{Low} \), \( S_{Med} \) and \( S_{High} \) shown in Figure 2A. These membership functions suggest typical values for each of the three fuzzy sets. We might select $6 million as the typical value for \( S_{Low} \), $8 million for \( S_{Med} \) and $10 million for \( S_{High} \). We compute a crisp value for \( S \) as the weighted average of these typical values, using the degrees of membership as weights:

\[
  S_{Crisp} = \frac{(0.32)(10) + (0.65)(8) + (0.12)(6) + (0.20)(6)}{0.32 + 0.65 + 0.12 + 0.20} = \frac{10.32}{1.29} = 8
\]
Thus, the executive would forecast quarterly sales of $8 million for the next quarter.

REFERENCES


